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Finite Element Beam/Plate Model for Modal Analysis of Light Aircraft Structures

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Abstract— *In the present work, the finite element method is adopted for modal analysis of a reduced model of light aircraft structures using Beam/Plate Model. The Aircraft structure is modeled by spatial beam and plate elements, where the fuselage is a long beam of isotropic constant section, wing and empennage (horizontal and vertical tails) are modeled by trapezoidal plate elements. The Lagrange principle is adopted to determine the mass matrix and stiffness matrix of both beams and plates. After assembling the elements in global mass and global stiffness matrices of the whole structure (fuselage, wings and empennage) followed by the introduction of the boundary conditions, a numerical method to determine the natural frequencies and associated modes is used. The obtained results are compared with those obtained by SAP 2000 "Structural Software for Analysis and design".*

Index Terms—FEM Modeling, Aircraft Structures, Modal Analysis, Beams, Plates

I. INTRODUCTION

The main components of an aircraft structure are wings, fuselages, tail units and control surfaces. The main functions of an aircraft's structure are to transmit and resist the applied loads, to provide an aerodynamic shape and to protect passengers, payload, systems, etc[5].

Finite element analysis (FEA) is a method for numerical solution of field problems. The applications of the FEA are : Structural, Vibration, fatigue and thermal[8]. Varghese and Kandhan [9]studied the flutter analysis of an aircraft wing and describe the triangle of forces of the aero elastic phenomena (aero elastic force, inertia force and elastic force).

A numerical procedure for the dynamic analysis of bi-dimensional plate elements formed in three-dimensional space described by Tizzi [6] is applied to aircraft structures.

In this paper, we are interest to the free vibration analysis of an aircraft structure model using the classical finite element approach. The finite element model of the symmetric one-half of the aircraft constructed by space beams and plates elements (Fig. 1) is used for free vibration analysis, this model is given by Gupta as a test model for aero-structural-control systems analysis using STARS[1].

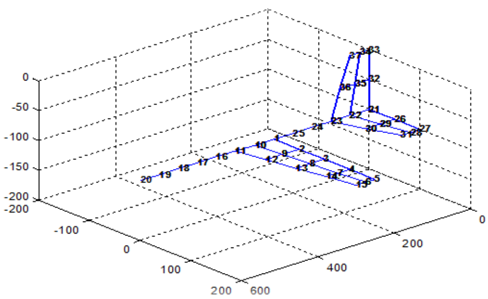

Figure 1. Aircraft Beam/Plate model

II. FINITE ELEMENT MODELLING

Modal analysis (natural frequencies and associated mode shapes) of an aircraft is an important step in its design. However, the matrix equation of vibration requires obtaining matrices mass and stiffness of the structure as the physical characteristics of materials. Therefore, the finite element method is adopted to model the light aircraft structure and deduce its mass and stiffness matrices.

A. The Fuselage Modeling

The fuselage is modeled by a collection of beams with tow nodes and six degrees of freedom in each node, where the sections of the beams are relatively constants and equal. The finite element beam presented in this section to idealize the fuselage is the Euler-Bernoulli beam which has a density ρ_b, modulus of elasticity E_b, shear modulus of elasticity G, cross-sectional area A_b, moment of inertia I_b, polar moment of inertia I_p, and length l_b.

The victor displacement of the element of the beam is given as:

$$\{q\} = \{u_1\ v_1\ w_1\ \theta x_1\ \theta y_1\ \theta z_1\ u_2\ v_2\ w_2\ \theta x_2 \theta y_2\ \theta z_2\}^t \tag{1}$$

A.1 Beam Mass Matrix

The finite element beam mass matrix for bending, torsion and linear bar are given by[2][3][4].

A.1.1 Bending Matrix

$$\left[M^e\right] = \frac{\rho_b A_b l_b}{420}\begin{bmatrix} 156 & 22l_b & 54 & 13l_b \\ & 4l^2 & 13l_b & -3{l_b}^2 \\ & & 156 & 22l_b \\ sym & & & 4{l_b}^2 \end{bmatrix} \tag{2}$$

A.1.2 Torsion Matrix

$$\left[M^e\right]=\frac{\rho_b A_b l_b}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} \tag{3}$$

A.1.3 Linear bar element

$$\left[M^e\right]=\frac{\rho_b A_b l_b}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} \tag{4}$$

A.2 Beam Stiffness Matrix

The finite element beam stiffness matrices for bending, torsion and linear bar are given by[2][3][4].

A.2.1 Bending Matrix

$$\left[K^e\right]=\frac{E_b I_b}{l_b^{\,3}}\begin{bmatrix}12 & 6l & -12 & 6l\\ & 4l^2 & -6l & 2l^2\\ & & 12 & -6l\\ sym & & & 4l^2\end{bmatrix} \tag{5}$$

A.2.2 Torsion Matrix

$$\left[K^e\right]=\frac{GI_p}{l_b}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \tag{6}$$

A.2.3 Linear bar element

$$\left[K^e\right]=\frac{E_b A_b}{l_b}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \tag{7}$$

B. The Wing and Tail Modeling

The finite element trapezoidal plate is used to model the aircraft's wing and tail. The plate element shown in fig. 2 has four nodes, each node of the element possesses three degrees of freedom : displacement *w*, rotation about the *x*-axis and rotation about the *y*-axis.

$$\theta_x=\frac{dw}{dy}\qquad \theta_y=\frac{dw}{dx} \tag{8}$$

The victor displacement of an element of trapezoidal plate is given as :

$$\{q\}=\{w_1\ \theta x_1\ \theta y_1\ w_2\ \theta x_2\ \theta y_2\ w_3\ \theta x_3\ \theta y_3\ w_4\ \theta x_4\ \theta y_4\}^t \tag{9}$$

The displacement function *w* assumed to be [7]:

$$w(x,y)=\alpha_1+\alpha_2 x+\alpha_3 y+\alpha_4 x^2+\alpha_5 xy+\alpha_6 y_3^2+\alpha_7 x^3+\alpha_8 x^2 y+\alpha_9 xy^2+\alpha_{10} y^3+\alpha_{11} x^3 y+\alpha_{12} xy^3 \tag{10}$$

this function can be written in natural coordinate system as:

$$w(\xi,\eta)=\sum_{i=1}^{4}\frac{1}{8}\left(1+\xi\xi_i\right)\left(1+\eta\eta_i\right)\left(2+\xi\xi_i+\eta\eta_i-\xi^2\eta^2\right)w_i+\sum_{i=1}^{4}\frac{b}{16}\eta_i\left(1+\xi\xi_i\right)\left(1+\eta\eta_i\right)^2\left(\eta\eta_i-1\right)\left(\frac{\partial w}{\partial y}\right)-\sum_{i=1}^{4}\frac{a}{16}\zeta_i\left(1+\xi\xi_i\right)^2\left(1+\eta\eta_i\right)\left(\xi\xi_i-1\right)\left(\frac{\partial w}{\partial x}\right) \tag{11}$$

where :

$$\zeta=2\left(x-x_0\right)/a\ ;\eta=\left(y-y_0\right)/b$$

$\left(x_0,y_0\right)$: the centre of plate coordinates.

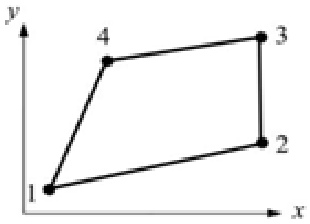

Figure 2. A trapezoidal plate element

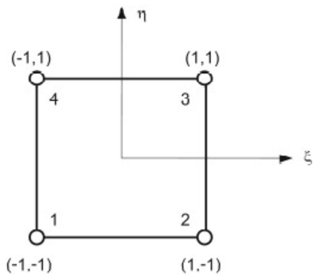

Figure 3. Plate element in natural coordinates

B.1 The Trapezoidal Plate Mass Matrix

The mass matrix of trapezoidal plate element is given by[7]:

$$\left[M^e\right]_p=\rho h\iint\left[N\right]^T\left[N\right]dx\,dy \tag{12}$$

where ρ is the plate density and *h* is the plate thickness. Equation(12) can be written in natural coordinates as:

$$\left[M^e\right]_p=\rho h\int_{-1}^{+1}\int_{-1}^{+1}\left[N\right]^T\left[N\right]\ J\,d\zeta\,d\eta \tag{13}$$

J is the Jacobian given by:

$$J=\frac{\partial\zeta}{\partial x}\frac{\partial\eta}{\partial y}-\frac{\partial\zeta}{\partial y}\frac{\partial\eta}{\partial x} \tag{14}$$

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B.2 The Trapezoidal Plate Stiffness Matrix.

The stiffness matrix of trapezoidal plate element is given by[7]:

$$[K^e]_p = \iint_s [B(x,y)]^t [D_z][B(x,y)]\,dx\,dy \qquad (15)$$

$$[D_z] = \frac{h^3}{12}[D]$$

E is Young's modulus, ν is Poisson's coefficient.

$$[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}$$

The equation(15) can be written in natural coordinates as:

$$[K^e]_p = \int_{-1}^{+1}\int_{-1}^{+1} [B(\zeta,\eta)]^t\,[D_z]\quad[B(\zeta,\eta)]\,J\,d\zeta\,d\eta \quad (16)$$

C. Assemblage and Modal Analysis

Once the mass and stiffness matrices of the each element are known, we can assemble and compute the corresponding matrices of the aircraft's structural model[6].

The matrix equation for vibration modal analysis of the whole aircraft's structure (beams and plates) is given (without damping) by[2][3][4][6]:

$$[M]\{\ddot{q}\} + [K]\{q\} = \{0\} \qquad (17)$$

$[M]$ and $[K]$ are the global masse and stiffness matrices of the whole structure.

III. AIRCRAFT MODEL

The one-half aircraft model is formed by space beams with trapezoidal plate elements(see, Gupta [1]).

The fuselage is a long isotropic constant section beam of 15.24 m.

The wing, which formed by trapezoidal element plates, is 2.54 m of length, 1.27 m of chord line of root profile and 0.635 m of tip line of profile.

The tail is 1.27 m height and length (see Fig.1).

The reduced aircraft model is fixed at two distinct points and the results obtained are compared with those obtained by SAP2000 software.

The characteristics of the material used are given in Table I.

TABLE I.
THE CHARACTERISTICS OF THE MATERIAL

Young's modulus	*71 GPa*
density	*2750 kg/m^3*
Poisson's coefficient	0.3

A. Aircraft model 1

First the one-half aircraft is fixed to the point of the rear end (the tail). The first natural frequencies and their associated mode shapes are given in Table II. The obtained results of the first model are graphically presented in fig.4 to fig. 11.

TABLE II.
FIRST FREQUENCIES OF AIRCRAFT REDUCED MODEL 1

Aircraft modes	**Present *(Hz)***	**SAP2000 *(Hz)***
1B fuse, 1B wing	03.8121	03.7796
2B fuselage	03.8555	03.8103
2B wing	05.0059	04.7528
1B emph, 1B empv	19.9043	18.4341
2B emph, 2B empv	20.0156	18.5068
3B fuse, 3B wing	22.8971	21.7274
4B fuselage	23.1747	22.4980
4B wing	24.1054	25.4276
1T wing	28.1733	28.0533
4B fuselage	55.0637	51.5055
5B wing	57.1481	60.6381
1T emph	57.5449	61.6099

B: bending, **T**: torsion, **emph**: horizontal empennage, **empv**: vertical empennage

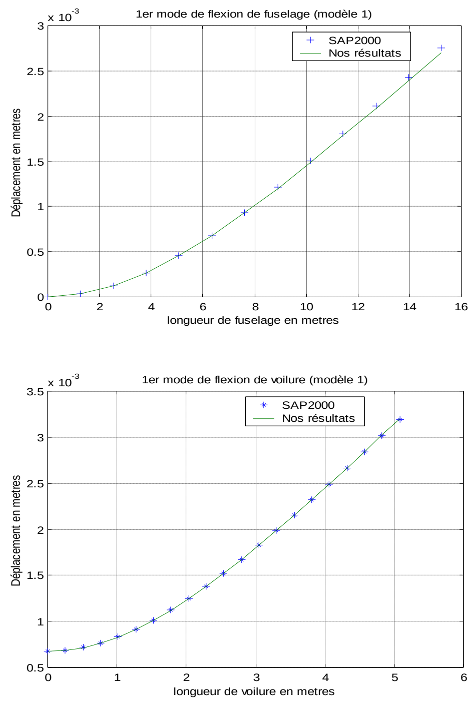


Figure 4. 1st mode shape of Aircraft Beam/Plate model1

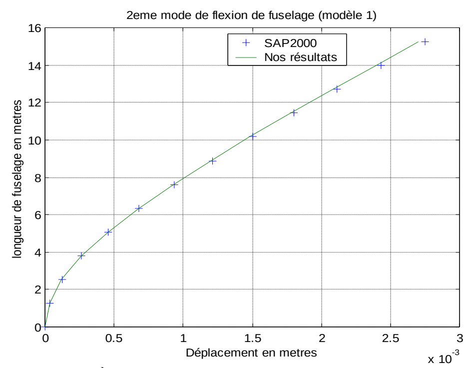


Figure 5. 2nd mode shape of Aircraft Beam/Plate model1

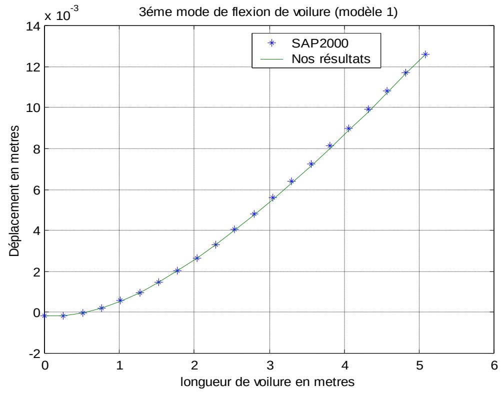
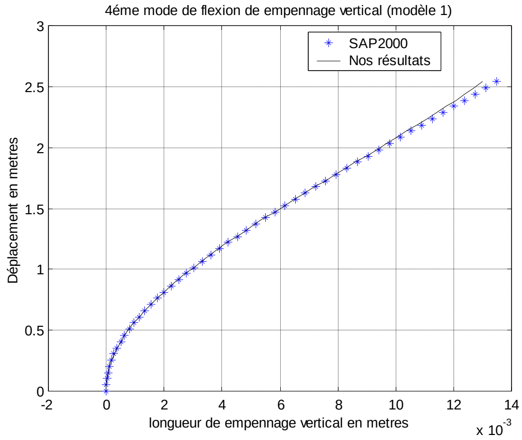


Figure 6. 3rd mode shape of Aircraft Beam/Plate model1



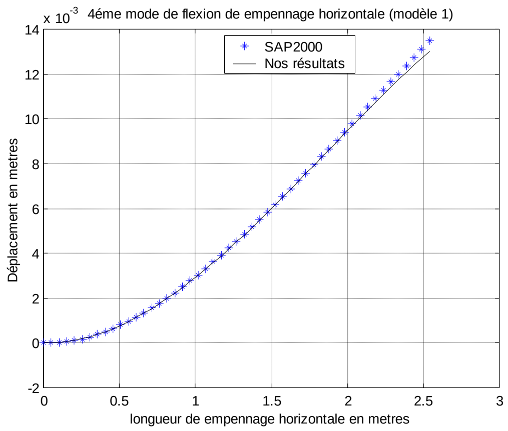
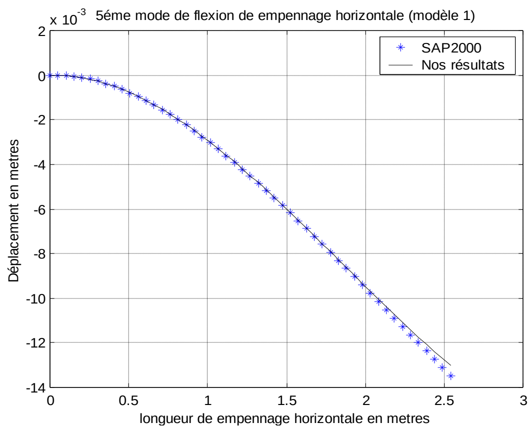


Figure 7. 4th mode shape of Aircraft Beam/Plate model1



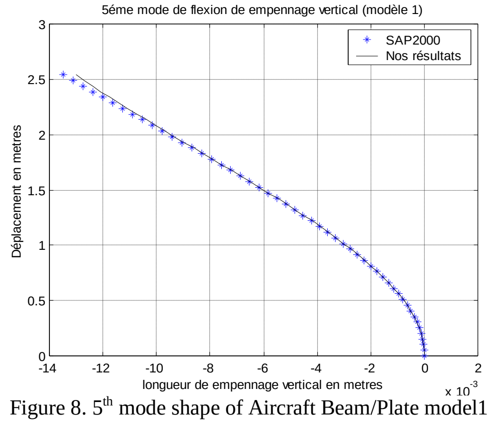
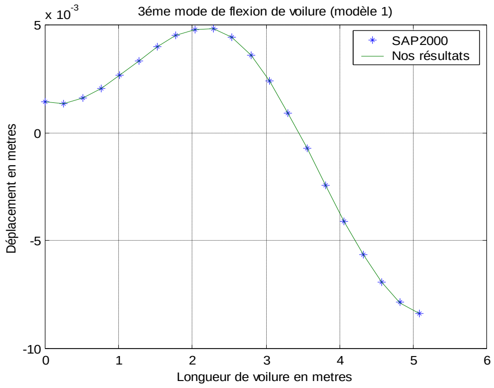


Figure 8. 5th mode shape of Aircraft Beam/Plate model1



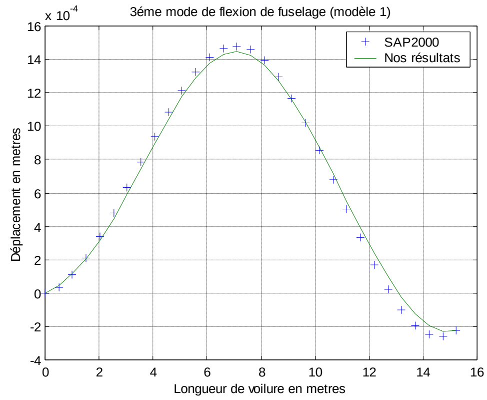


Figure 9. 6th mode shape of Aircraft Beam/Plate model1

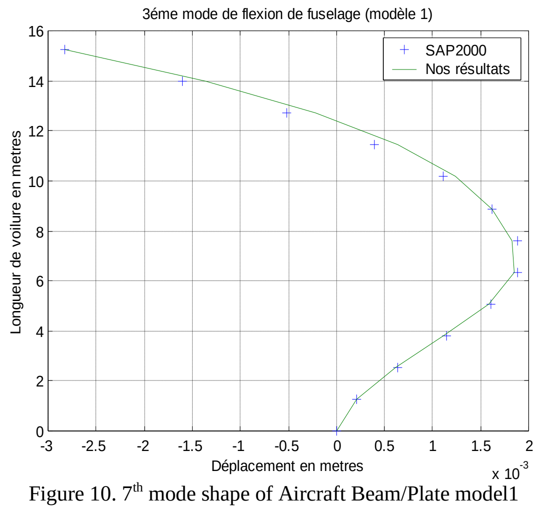


Figure 10. 7th mode shape of Aircraft Beam/Plate model1

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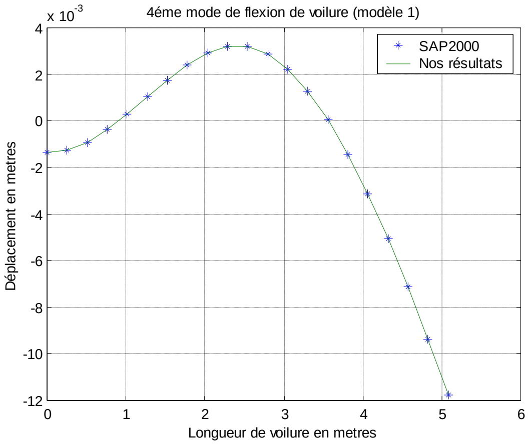

Figure 11. 8th mode shape of Aircraft Beam/Plate model1

B. Aircraft model 2

In second aircraft reduced model, the One-half plane is fixed in its center of gravity, the first five mode shapes are given in Table III.

TABLE III.
FIRST FREQUENCIES OF AIRCRAFT REDUCED MODEL2

Aircraft modes	**Present *(Hz)***	**SAP2000 *(Hz)***
1B wing	04.9585	04.7156
1B fuselage	08.5838	08.5134
2B fuselage	08.6871	08.5857
1B emph, empv	19.1009	17.6059
2B emph, empv	19.5050	17.9588
2B wing	24.1035	23.9347
1T wing	27.1580	28.0235
3B fuselage	30.2394	29.4368
4B fuselage	30.2704	29.4688
5B fuselage	48.6146	45.8879
6B fuselage	53.3432	50.5116
3B wing	57.4291	61.2568

B: bending, **T**: torsion, **emph**: horizontal empennage, **empv**: vertical empennage

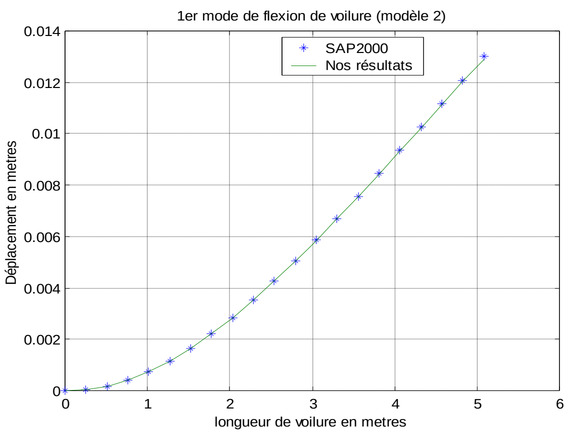

Fig. 12. 1st mode shape of Aircraft Beam/Plate model2

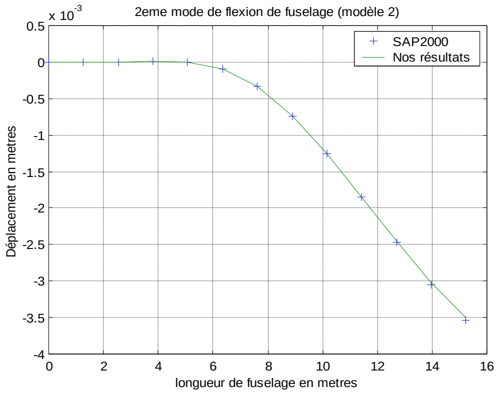

Figure. 13. 2nd mode shape of Aircraft Beam/Plate model2

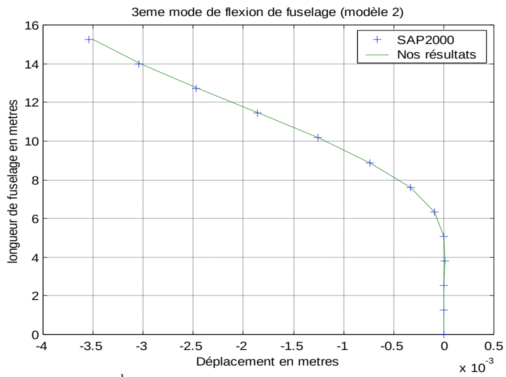

Figure 14. 3rd mode shape of Aircraft Beam/Plate model2

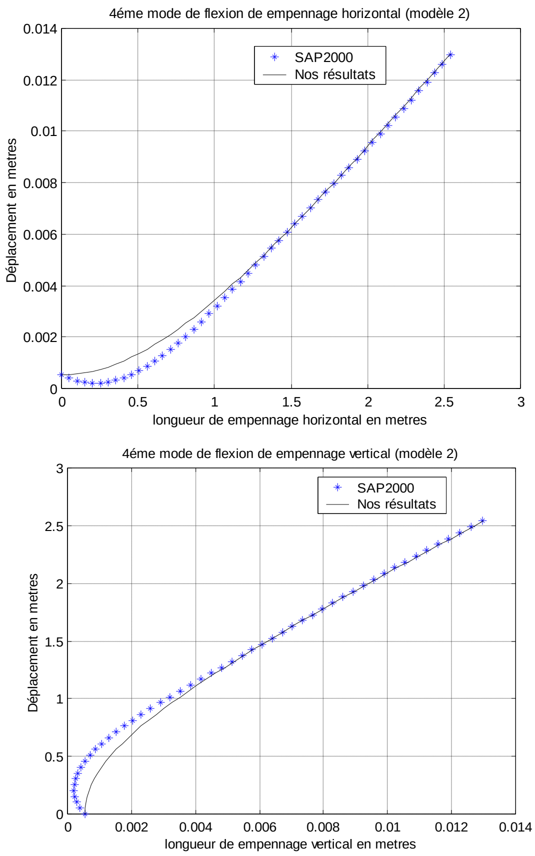

Figure 15. 4th mode shape of Aircraft Beam/Plate model2

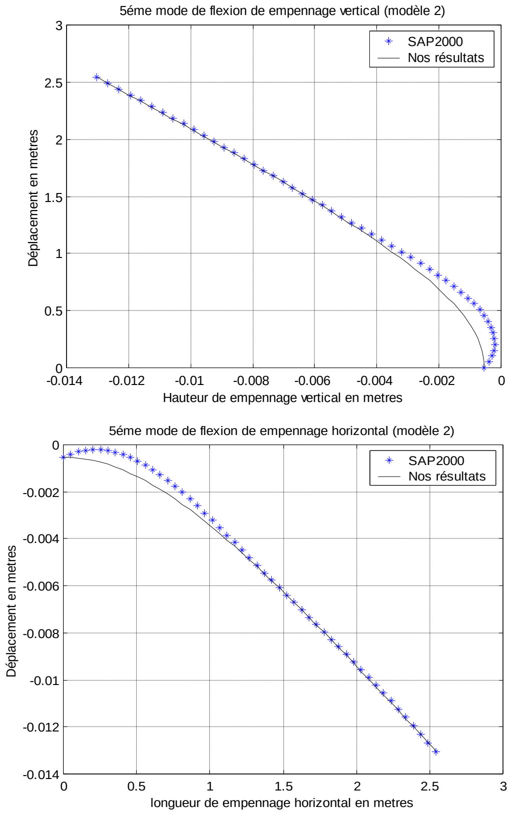

Figure 16. 5th mode shape of Aircraft Beam/Plate model2

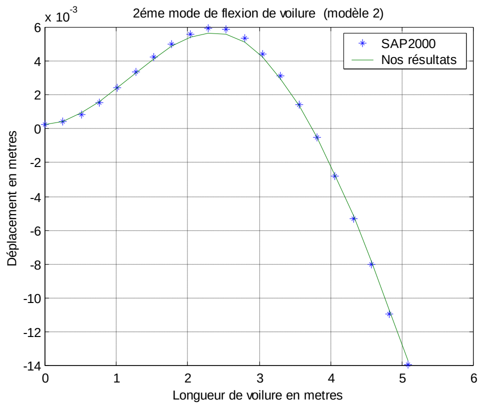

Figure 17. 6th mode shape of Aircraft Beam/Plate model2

C. concluding remarks

It's appears clearly, from the results obtained for two one-half aircraft models that the obtained frequencies and their associated mode shapes are very closely to those obtained by SAP software. It's remarked that changing of the support point affects the mode shapes and natural frequencies.

The natural frequencies and mode shapes of the first model (support point in the rear of plane) show that the first mode shape is bending of both fuselage and wing. However, the results for the second model (support point in the centre of gravity of plane) show that the first mode shape is the bending of the wing alone.

The four first modes of the first model are bending modes where the torsion mode of wing appears in the second model before the third fuselage bending mode. Two or more substructures modes can be visualized in one aircraft mode.

CONCLUSIONS

Modeling of reduced light aircraft structures by using Beam/Plate finite element one-half model for free vibration analysis is presented in this paper. The fuselage as long beam, the wing and tail as trapezoidal plates is a simplified model of aircraft's structure, the idea is to show the behavior of the structure using two proposed supported points which help us and researchers to go further studies in dynamics, aero elasticity, stability and control. The presented model will be generalized to light aircrafts in composite wings, thick plates and Timoshenko beams where the effect of shear is important.

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